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ELECTROSTATIC SOLITARY WAVE STRUCTURES IN MAGNETIZED NEGATIVE ION PLASMA WITH STATIONARY DUST

*N. AKHTAR and S. HUSSAIN[1]

Theoretical Plasma Physics Division (TPPD), PINSTECH, P.O. Nilore, Islamabad, Pakistan

[1]Department of Physics and Applied Mathematics (DPAM), PIEAS, P.O. Nilore, Islamabad, Pakistan

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Non linear electrostatic solitary wave structures are studied in negative ion magnetized homogenous plasma in the presence of stationary dust. Zakharov-Kuznetsov (ZK) equation is derived by using reductive perturbation method. Laboratory plasma parameters are used for the numerical analysis of ZK equation. It is seen that compressive and rarefactive solitary wave structures are observed for fast and slow modes. The effects of magnetic field, obliqueness, temperature variation of positive and negative ions in the presence of dust on non linear solitary wave structures are discussed.

Keywords : Electrostatic solitary waves, Magnetized plasma, Negative plasma, ZK equation, Reductive perturbation method.

1. Introduction

Negative ion plasma plays an important role in the study of different fields of the plasma physics. This type of plasma consists of negative ions, positive ions and electrons. The occurrence of negative ion plasma in space and in astrophysical environments is well known [1, 2]. There are many examples of negative ion plasma systems i.e., neutral beam sources [3], photosphere of the Sun [4], D region of the ionosphere [5] and plasma processing reactors [6] etc. Negative ions are produced by the electrons attachment to the neutral atoms or molecules. These negative ions change the plasma potential and electrons behavior [7-9]. There are several methods to produce negative ion plasma in laboratory. In a gas discharge device iodine gas at high pressures has been used to produce negative ion plasma with very few electrons. On the other hand the addition of a hexafluoride gas such as SF_6, to a positive ion-electron plasma is used to produce a controllable concentration of negative ions in a gas discharge device [10]. The importance of negative ion plasma is also increasing due to its surpass positive ions in plasma etching [11-13].

Negative ion plasmas observed in laboratory and in space are not pure due to presence of dust particles in general [14]. The dust particles in the plasmas are charged due to plasma currents, ultraviolet irradiation, field emission and electron attachment etc. Merlino et al. [4] explained the charging and decharging of dust grains experimentally. It has been experimentally as well as theoretically explained that the presence of dust in the negative ion plasma modifies the characteristic behavior of the plasma waves even at frequencies where the dust grains do not participate in the wave motion [15-21]. There are numbers of eigen modes which exist due to presence of dust in the plasma e.g. dust ion-acoustic (DIA) waves, dust-acoustic (DA) waves, dust-lattice (DL) waves, dust-lower-hybrid waves, dust ion--cyclotron waves, dust--cyclotron (DC) waves, Shukla--Varma (SV) mode, dust shear-Alfv´en (DSA) waves and dust-magnetoacoustic (DM) waves etc. [22]. Merlino et al. [23] had made theoretical and experimental investigations on low frequency dust ion acoustic waves. They showed that phase velocity of the DIA wave increases with increasing relative dust concentration in the plasmas containing positive and negative ions in the presence of stationary dust.

It is always interesting to study the ion acoustic solitary wave structures in the presence of magnetic field. The behavior of plasma particle changes significantly with the induction of the magnetic field in the plasma system [24,25]. Mahmood and Akhtar [26] studied the ion acoustic solitary wave structure in multicomponent magnetized plasma with adiabatically heated ions. They explained the variation in the solitary wave structure in the presence of magnetic field in the

* Corresponding author : naseemqau@gmail.com

pulsars cusp regions. Kourakis et al. [27] derived the ZK equation to study the rotational effect on the formation of multidimensional solitons in e-p-i and pair ion plasmas (a type of negative ion plasma). Mushtaq et al. [28] studied the ion acoustic waves in magnetized pair ion electron plasmas. They derived ZK equation for pair ion plasmas and revealed the fact that nonlinear profile of the IA solitary wave is significantly affected by the obliqueness, magnetic field and the electron concentration.

We have studied the small amplitude ion acoustic solitary wave in negative ion magnetized plasma containing stationary dust. ZK equation is derived by using reduction perturbation method. The effects of different plasma parameters on propagation of the solitary wave structure are discussed. The paper is organized in the following way. In the next section, we will describe the mathematical model and set of governing equations for the negative ion magnetized plasmas. In Sec. 3, the ZK equation is derived by using the reductive perturbation method and soliton solution is described. In Sec. 4 the numerical plots are presented. Conclusions are presented in Sec.5.

2. Nonlinear Set of Equations

In this section, the electrostatic wave in negative, positive ions and electron in the homogenous magnetized plasmas has been derived in the presence of stationary dust. The fluids of negative and positive ions are assumed to be dynamic while electrons follow the Boltzmann distribution. The external magnetic field is taken along x-axis i.e., $B_0 = B_0 \hat{x}$. The set of equations for the dynamics of negative, positive ions in the magnetized plasmas are given by,

$$\frac{\partial n_j}{\partial t} + \nabla.\ n_j u_j \ = 0 \qquad (1)$$

$$\frac{\partial u_j}{\partial t} + \ u_j.\nabla\ u_j = \frac{q_j}{m_j}\left(E + \frac{u_j \times B_0 \hat{x}}{c}\right) - \frac{\nabla p_j}{m_j n_j} \qquad (2)$$

For adiabatic hot ions, we have

$$p_j = p_{j0}\left(\frac{n_j}{n_{j0}}\right)^{\gamma} \qquad (3)$$

where γ= (N+2)/N, and N is the number of degrees of freedom. The present work N=3 and hence γ=5/3 and $p_{j0} = n_{j0} T_j$

Electrons follow the Boltzmann distribution

$$n_e = n_{e0} \exp^{\frac{e\varphi}{T_e}} \qquad (4)$$

The Poisson equation can be written as follows

$$\nabla.E = 4\pi e\ \ n_+ - n_- - n_e - Z_d n_{d0} \qquad (5)$$

In the presence of dust particles the equilibrium condition can be written as

$$n_{+0} = n_{-0} + n_{e0} + Z_d n_{d0} \qquad (6)$$

where the electric field intensity and perturbed densities of the positive, negative ions and electron are E, n_+, n_- and n_e respectively. The equilibrium densities of positive, negative ions, electrons, and dust are n_{+0}, n_{-0}, n_{e0} and n_{d0} respectively. The masses of positive and negative ions are (m_+, m_-) and are oppositely charged i.e., +e and -e respectively.

Now assuming the wave propagation in two dimension i.e.,$\nabla = \ \partial_x, \partial_y, 0$. The normalized [29-32] continuity and momentum equations for positive ions in the component form can be written as,

$$\frac{\partial n_+}{\partial t} + \frac{\partial\ \ n_+ u_{+x}}{\partial x} + \frac{\partial\ \ n_+ u_{+y}}{\partial y} = 0 \qquad (7)$$

$$\frac{\partial u_{+x}}{\partial t} + \left(u_{+x}\frac{\partial}{\partial x} + u_{+y}\frac{\partial}{\partial y}\right)u_{+x} = -\frac{\partial \Phi}{\partial x} - \frac{\alpha}{n_+}\frac{\partial n_+^{\frac{5}{3}}}{\partial x} \qquad (8)$$

$$\frac{\partial u_{+y}}{\partial t} + \left(u_{+x}\frac{\partial}{\partial x} + u_{+y}\frac{\partial}{\partial y}\right)u_{+y} = -\frac{\partial \Phi}{\partial y} - \Omega u_{+z} - \frac{\alpha}{n_+}\frac{\partial n_+^{\frac{5}{3}}}{\partial y} \qquad (9)$$

$$\frac{\partial u_{+z}}{\partial t} + \left(u_{+x}\frac{\partial}{\partial x} + u_{+y}\frac{\partial}{\partial y}\right)u_{+z} = -\Omega u_{+y} \qquad (10)$$

The normalized continuity and momentum equations for negative ions in the component form can be written as,

$$\frac{\partial n_-}{\partial t}+\frac{\partial\ n_-u_{-x}}{\partial x}+\frac{\partial\ n_-u_{-y}}{\partial y}=0 \qquad (11)$$

$$\frac{\partial u_{-x}}{\partial t}+\left(u_{-x}\frac{\partial}{\partial x}+u_{-y}\frac{\partial}{\partial y}\right)u_{-x}=\delta\left(\frac{\partial\Phi}{\partial x}-\frac{\beta}{n_-}\frac{\partial n_-^{\frac{5}{3}}}{\partial x}\right) \qquad (12)$$

$$\frac{\partial u_{-y}}{\partial t}+\left(u_{-x}\frac{\partial}{\partial x}+u_{-y}\frac{\partial}{\partial y}\right)u_{-y}=\delta\left(\frac{\partial\Phi}{\partial y}-\Omega u_{+z}-\frac{\beta}{n_-}\frac{\partial n_-^{\frac{5}{3}}}{\partial y}\right) \qquad (13)$$

$$\frac{\partial u_{-z}}{\partial t}+\left(u_{-x}\frac{\partial}{\partial x}+u_{-y}\frac{\partial}{\partial y}\right)u_{-z}=-\delta\Omega u_{-y} \qquad (14)$$

Electron follows the Boltzmann distribution

$$n_e=\exp\ \Phi \qquad (15)$$

The Poisson equation in the normalized form is given by

$$\nabla^2\Phi=n_+-Pn_--Qn_e-\ 1-P-Q \qquad (16)$$

where the electric field is defined as $E=-\nabla\varphi$ (where φ is the electrostatic potential). The normalization $t\to t\omega_{p+}$, $\nabla\to\frac{\nabla}{\lambda_{De}}$, $n_j\to\frac{n_j}{n_{j0}}$, $u_j\to\frac{u_j}{C_s}$ and $\Phi\to\frac{e\varphi}{T_e}$ have been defined, where $\omega_{p+}=\sqrt{\left(\frac{4\pi n_{+0}e^2}{m_+}\right)}$ is the plasma frequency for positive ions and $\lambda_{De}=\sqrt{\left(\frac{T_e}{4\pi n_{+0}e^2}\right)}$ is the Debye length for electron. The dimension less parameters such as negative ion to positive ion equilibrium density ratio i.e., $P=\frac{n_{-0}}{n_{+0}}$, electron to positive ion equilibrium density ratio i.e., $Q=\frac{n_{e0}}{n_{+0}}$ has been defined. $\alpha=\frac{T_+}{T_e}$ is the positive ion temperature to the electron temperature, $\beta=\frac{T_-}{T_e}$ is the negative ion temperature to the electron temperature , the mass ratio of positive ion to negative ion is $\delta=\frac{m_+}{m_-}$. Cyclotron to positive ion plasma frequency ratio $\Omega=\left(\frac{\omega_{+c}}{\omega_{p+}}\right)$ has been defined, where $\omega_{+c}=\left(\frac{eB_0}{m_+c}\right)$ is the positive ion gyro frequency.

3. Nonlinear Solution

In order to find the Zakharov-Kuznetsov (ZK) equation for electrostatic potential in a homogeneous magnetized negative ion plasma in the presence of stationary dust, we define the stretching of independent variable such as Washimi and Tanuiti [33],

$$X=\varepsilon^{1/2}\ l_x x-\lambda t$$
$$Y=\varepsilon^{1/2}l_y y\ ,$$
$$\tau=\varepsilon^{3/2}t \qquad (17)$$

where ε is a small $0<\varepsilon\le 1$ expansion parameter characterizing the strength of the nonlinearity and λ is the phase velocity of the wave normalized with thermal velocity of electron. Here l_x and l_y are direction cosines respectively such that $l_x^{\ 2}+l_y^{\ 2}=1$.

Now using the reductive perturbation method, we can expand the perturbed quantities (Khan and Masood [34] Kourakis et al. [35]) about their equilibrium values in the powers of ε as follows,

$$n_j=1+\varepsilon n_j^{\ 1}+\varepsilon n_j^{\ 2}+\varepsilon n_j^{\ 3}+...$$

$$u_{jx}=\varepsilon u_{jx}^{\ 1}+\varepsilon^2 u_{jx}^{\ 2}+\varepsilon^3 u_{jx}^{\ 3}+...$$

$$u_{jy}=\varepsilon^2 u_{jy}^{\ 1}+\varepsilon^3 u_{jy}^{\ 2}+\varepsilon^4 u_{jy}^{\ 3}+... \qquad (18)$$

$$u_{jz}=\varepsilon^{3/2}u_{jz}^{\ 1}+\varepsilon^{5/2}u_{jz}^{\ 2}+\varepsilon^{7/2}u_{jz}^{\ 3}+...$$

$$\Phi=\varepsilon\Phi^{\ 1}+\varepsilon^2\Phi^{\ 2}+\varepsilon^3\Phi^{\ 3}+...$$

Using Eqs.(17-18) in Eqs.(7-16) and collecting terms of lowest order ($\sim\varepsilon^{3/2}$) of continuity and momentum equations of positive and negative ions, we have

$$n_+^{\ 1}=-\frac{3l_x^{\ 2}\Phi^{\ 1}}{5\alpha l_x^{\ 2}-3\lambda^2}$$

$$u_{+x}^{\ 1}=-\frac{3\lambda l_x\Phi^1}{5\alpha l_x^{\ 2}-3\lambda^2} \tag{19}$$

$$u_{+z}^{\ 1}=-\frac{l_y}{\Omega}\frac{3\lambda^2}{5\alpha l_x^{\ 2}-3\lambda^2}\frac{\partial\Phi^1}{\partial Y}$$

$$n_-^{\ 1}=\frac{3\delta l_x^{\ 2}\Phi^1}{5\beta\delta l_x^{\ 2}-3\lambda^2}$$

$$u_{-x}^{\ 1}=\frac{3\delta\lambda l_x\Phi^1}{5\beta\delta l_x^{\ 2}-3\lambda^2} \tag{20}$$

$$u_{-z}^{\ 1}=\frac{l_y}{\Omega}\frac{3\lambda^2}{5\beta\delta l_x^{\ 2}-3\lambda^2}\frac{\partial\Phi^1}{\partial Y}$$

The lowest order ($\sim\varepsilon$) term from the electron and Poisson equations give

$$n_e^{\ 1}=\Phi^1 \tag{21}$$

$$n_+^{\ 1}-Pn_-^{\ 1}-Qn_e^{\ 1}=0 \tag{22}$$

Now using the expressions of $n_+^{\ 1}$, $n_-^{\ 1}$ and $n_e^{\ 1}$ from Eqs.(19-21) in Eq.(22), the linear phase speed of the acoustic wave is obtained as follows,

$$\lambda^2=\frac{L\pm\sqrt{L^2-M}}{18Q} \tag{23}$$

Where $L=9l_x^{\ 2}\ 1+P\delta\ +15Ql_x^{\ 2}\ \alpha+\beta\delta$ and $M=180Q\delta\ 3\ \beta+\alpha P\ +5Q\alpha\beta$ are defined in the following way.

Both (±) fast and slow modes have been obtained in negative ion plasma containing stationary dust.

Now collecting the next higher order terms i.e., ($\varepsilon^{5/2}$) from continuity and the momentum equations of positive ions, negative ions and Poisson's equation and after some mathematical manipulations (see appendix) we obtain the Zakharov-Kuznetsov (ZK) equation for electrostatic wave in magnetized negative plasmas in the presence of stationary dust in terms of Φ^1 as follows,

$$\partial_\tau\Phi^1+A\Phi^1\partial_X\Phi^1+\partial_X\ B\partial_X^{\ 2}+C\partial_Y^{\ 2}\ \Phi^1=0 \tag{24}$$

here the nonlinear coefficient A and the dispersive coefficients B and C are defined as,

$$A=\left(\frac{A1+A2+A3}{Denom}\right)$$

$$B=\left(\frac{-l_x^{\ 2}\ 5\alpha l_x^{\ 2}-3\lambda^2\ \ 5\beta\delta l_x^{\ 2}-3\lambda^2}{Denom}\right) \tag{25}$$

$$C=\left(\frac{C1+C2+C3}{Denom}\right)$$

Where Denom, A1, A2, A3, C1, C2, C3 are defined as

Demon=

$$\left(\frac{18\lambda l_x^{\ 2}\ 5\beta\delta l_x^{\ 2}-3\lambda^2}{5\alpha l_x^{\ 2}-3\lambda^2}-\frac{18P\delta\lambda l_x^{\ 6}\ 5\alpha l_x^{\ 2}-3\lambda^2}{5\beta\delta l_x^{\ 2}-3\lambda^2}\right)$$

A1= $\dfrac{81P\lambda^2\delta^2 l_x^{\ 4}\ 5\alpha l_x^{\ 2}-3\lambda^2}{5\beta\delta l_x^{\ 2}-3\lambda^2\ {}^2}-\dfrac{81\lambda^2 l_x^{\ 4}}{5\alpha l_x^{\ 2}-3\lambda^2\ {}^2}$

A2= $-\ 5\alpha l_x^{\ 2}-3\lambda^2\ \ 5\beta\delta l_x^{\ 2}-3\lambda^2\ Q$

A3=

$$\frac{15\alpha l_x^{\ 6}\ 5\beta\delta l_x^{\ 2}-3\lambda^2}{5\alpha l_x^{\ 2}-3\lambda^2\ {}^2}-\frac{15P\beta\delta^3 l_x^{\ 6}\ 5\alpha l_x^{\ 2}-3\lambda^2}{5\beta\delta l_x^{\ 2}-3\lambda^2\ {}^2}$$

C1= $\dfrac{9\lambda^4 l_y^{\ 2}\ 5\beta\delta l_x^{\ 2}-3\lambda^2}{\Omega^2\ 5\alpha l_x^{\ 2}-3\lambda^2}$

C2= $\dfrac{9P\lambda^4 l_y^{\ 2}\ 5\alpha l_x^{\ 2}-3\lambda^2}{\delta\Omega^2\ 5\beta\delta l_x^{\ 2}-3\lambda^2}$

C3= $-l_y^{\ 2}\ 5\alpha l_x^{\ 2}-3\lambda^2\ \ 5\beta\delta l_x^{\ 2}-3\lambda^2$

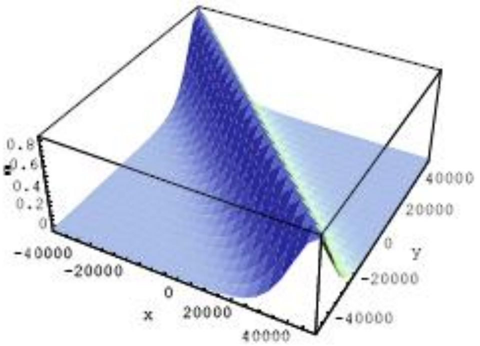
The ZK equation Eq.(24) has a well known soliton solution, which is given by

$$\Phi^1=\varphi_m\sec h^2\left(\frac{\eta}{W}\right)$$

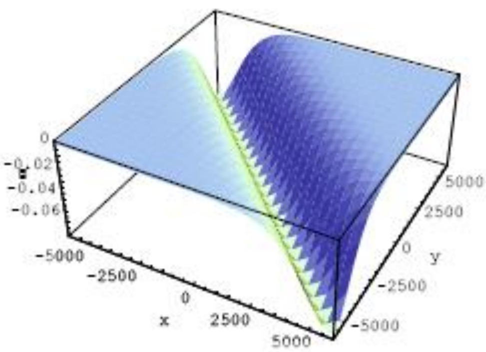
where $\varphi_m=\ 3u_0/A$ and $W=\sqrt{4\ B+C\ /u_0}$ are the amplitude and the width of the soliton, and η is the transformed coordinate in the co-moving frame with speed u_0 (i.e., $\eta=X+Y-u_0\tau\sum$). The coefficients of ZK equation contributes to the formation of amplitude φ_m, and width W, of the soliton that can easily be seen from the expression given in Eq.(24).

4. Analytical Solutions

In this section, we have studied the influence of different plasma parameters on propagation characteristics of solitary wave structures in magnetized negative ion plasma in the presence of stationary dust. By means of numerical computation, we have studied the effects of negative ion density, temperature of positive and negative ions, and magnetic field intensity by plotting electrostatics potential Φ^1 against η. We have used the values of laboratory plasma parameters (Takeuchi et al. [36]) as $n_{e0} \approx 10^9 cm^{-3}$, $n_{+0} \approx 10^9 cm^{-3}$, $n_{-0} \approx 10^9 cm^{-3}$, $T_+ = 1.1604\times10^4 K$, $T_- = 1.1604\times10^4 K$, $T_e = 1.1604\times10^5 K$, $B_0 = 25$ G and Carbon Hydrogen plasma in the numerical analysis of Eq. (24) for both fast and slow modes as shown in Figures 1a, and 1b. It is noted that the phase velocity, amplitude and width of the solitary wave structure is reduced for slow mode. Compressive solitary wave structures are observed for the fast mode and rarefactive are observed for the slow mode in the presence of dust particles in magnetized negative ion plasma. It is seen that by decreasing the value of magnetic field intensity results the amplification in the width of solitary wave structure for both modes as display in Figure 2a and 2b. It is also noted that the amplitude of solitary wave structure is reduced by

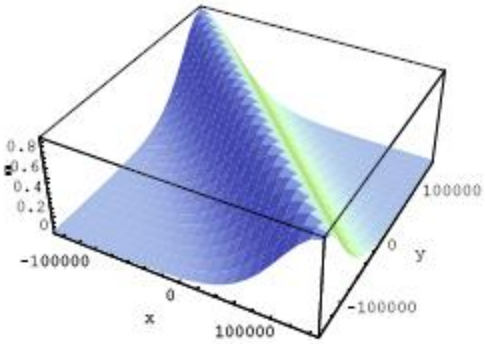


(a)

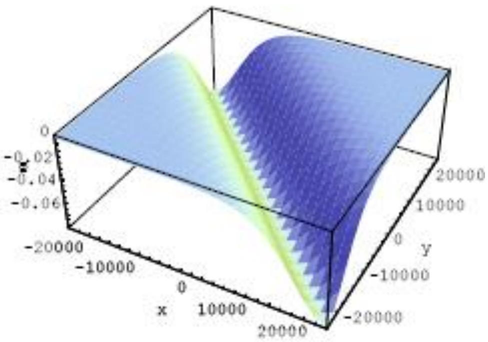


(b)

Figure 1. The electrostatic potential hump structure is shown for electron density $n_{e0} = 0.45\times10^9 cm^{-3}$ and positive ion density $n_{+0} = 1\times10^9 cm^{-3}$ negative ion density $n_{-0} = 0.45\times10^9 cm^{-3}$, with $T_e = 1.1604\times10^5 K$, $T_+ = 1.1604\times10^4 K$, $T_- = 1.1604\times10^4 K$, $l_x = 0.80$, $B_0 = 5$ Gauss and $u_0 = 0.06$ a (fast mode) b (slow mode).



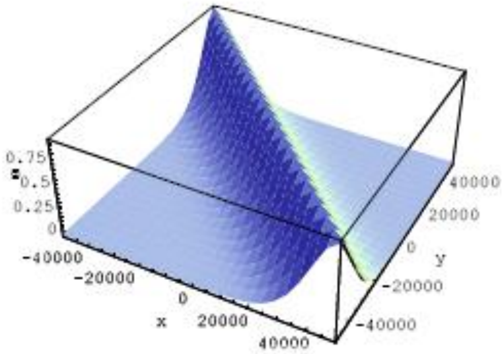
(a)



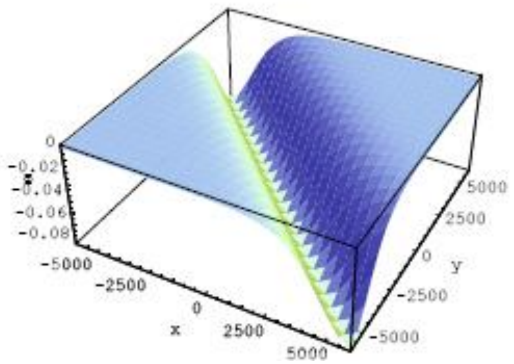
(b)

Figure 2 The amplification of soliton width with variation of magnetic field $n_{e0} = 0.45\times10^9 cm^{-3}$ and positive ion density $n_{+0} = 1\times10^9 cm^{-3}$ negative ion density $n_{-0} = 0.45\times10^9 cm^{-3}$, with $T_e = 1.1604\times10^5 K$, $T_+ = 1.1604\times10^4 K$, $T_- = 1.1604\times10^4 K$, $l_x = 0.80$, $B_0 = 1$ Gauss and $u_0 = 0.06$ a (fast mode) b (slow mode).

decreasing the value of magnetic field intensity. On the other hand by decreasing the value of direction cosine l_x the amplitude of the solitary structure augmented for both modes has observed by the variation of obliqueness as shown in Figure 3a and 3b. If we increase the temperature of positive ions in the plasma system then the value of α i.e. ratio of positive ions to electrons increases. It is observed that increment in α results reduction in the amplitude of the solitons for the fast mode and enhancement in the width of slow mode as shown.



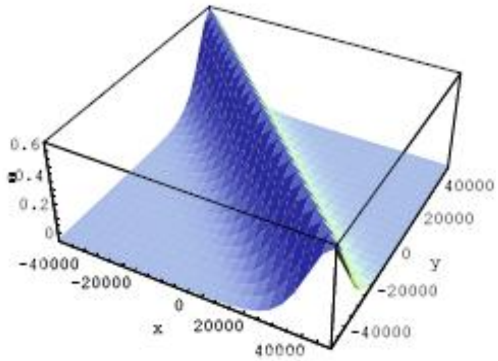
(a)



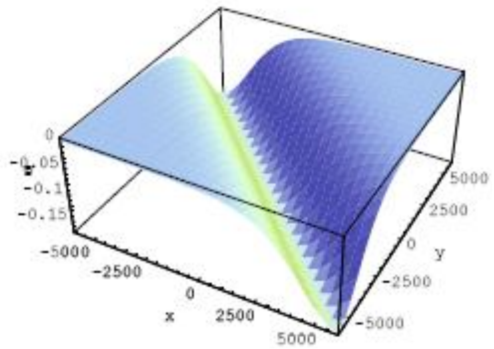
(b)

Figure 3. The effect of obliqueness for electron density $n_{e0} = 0.45 \times 10^9 \text{cm}^{-3}$ and positive ion density $n_{+0} = 1 \times 10^9 \text{cm}^{-3}$ negative ion density $n_{-0} = 0.45 \times 10^9 \text{cm}^{-3}$, with $T_e = 1.1604 \times 10^5$K, $T_+ = 1.1604 \times 10^4$K, $T_- = 1.1604 \times 10^4$K, $l_x = 0.75$, $B_0 = 5$ Gauss and $u_0 = 0.06$ a (fast mode) b (slow mode).

in Figure 4a and 4b. The variation in the temperature of negative ions is observed via parameter β. It is seen that by enhancing the value of β i.e. ratio of the temperature of negative ions to electrons, the amplitude of the solitary wave for fast mode structure is augmented. It is also noted that for slow mode the amplitude decrease and width increases significantly as shown in Figure 5a and 5b. It is observed that by increasing the value of mass ratio of positive to negative ions i.e. δ the amplitude decreases and width of solitary wave structure increases for both modes as shown in the Figure 6a and 6b. It is also examined that width of the solitary structures increases and amplitude decrease in the absence of dust particles in magnetized negative ion plasma for both fast and slow modes as depicts in Figure 7a and 7b.

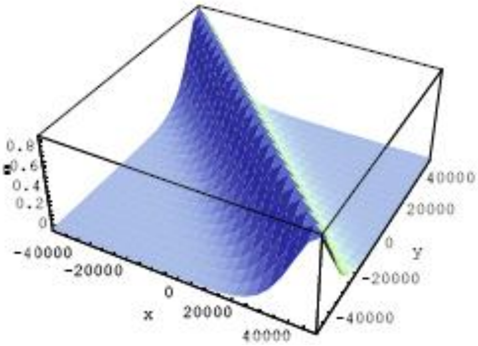


(a)

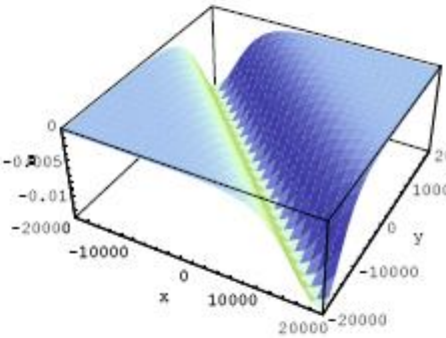


(b)

Figure 4. The reduction in the amplitude of solitons with increase in positive ion temperature. The electrostatic potential hump structures is shown for electron density $n_{e0} = 0.45 \times 10^9 \text{cm}^{-3}$ and positive ion density $n_{+0} = 1 \times 10^9 \text{cm}^{-3}$ negative ion density $n_{-0} = 0.45 \times 10^9 \text{cm}^{-3}$, with $T_e = 1.1604 \times 10^5$K, $T_+ = 3 \times 1.1604 \times 10^4$K, $T_- = 1.1604 \times 10^4$K, $l_x = 0.80$, $B_0 = 5$ Gauss and $u_0 = 0.06$ a (fast mode) b (slow mode).

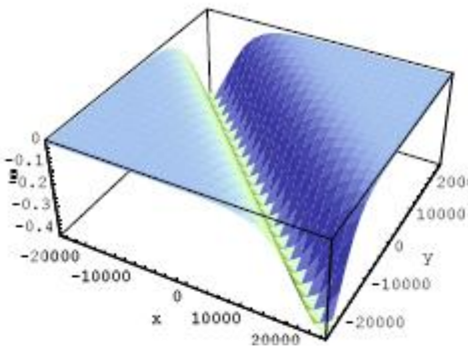


(a)

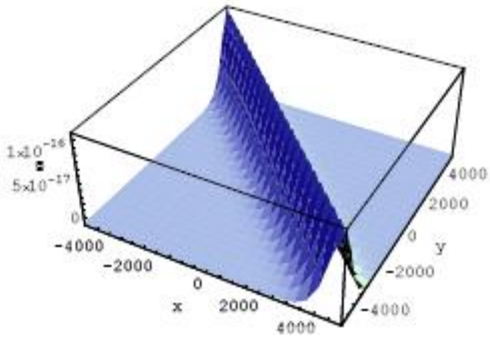


(b)

Figure 5. The electrostatic potential hump structures increment with rise in negative ion temperature is shown for electron density $n_{e0} = 0.45 \times 10^9 \text{cm}^{-3}$ and positive ion density $n_{+0} = 1 \times 10^9 \text{cm}^{-3}$ negative ion density $n_{-0} = 0.45 \times 10^9 \text{cm}^{-3}$ with $T_e = 1.1604 \times 10^5$K, $T_+ = 1.1604 \times 10^4$K, $T_- = 8 \times 1.1604 \times 10^4$K, $l_x = 0.80$, $B_0 = 5$ Gauss and $u_0 = 0.06$ a (fast mode) b (slow mode).

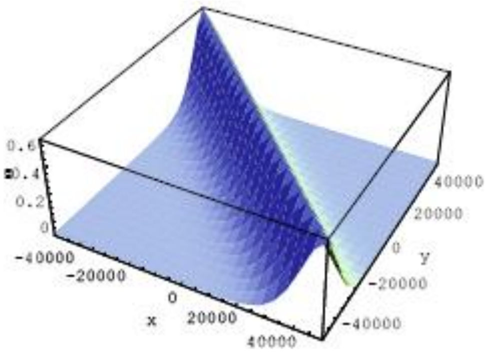


(a)

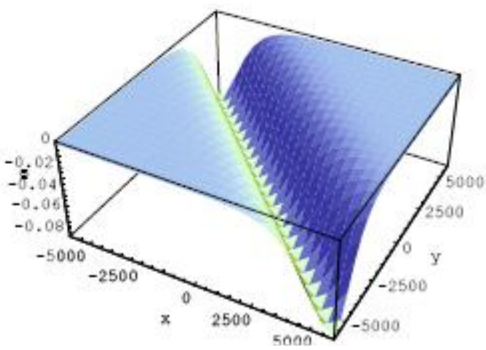


(b)

Figure 6. Variation of electrostatic potential with variation in δ is shown for electron density $n_{e0} = 0.45 \times 10^9 \text{cm}^{-3}$ and positive ion density $n_{+0} = 1 \times 10^9 \text{cm}^{-3}$ negative ion density $n_{-0} = 0.45 \times 10^9 \text{cm}^{-3}$ with $T_e = 1.1604 \times 10^5$K, $T_+ = 1.1604 \times 10^4$K, $T_- = 1.1604 \times 10^4$K, $l_x = 0.80$, $B_0 = 5$ Gauss and $u_0 = 0.06$ a (fast mode) b (slow mode).



(a)



(b)

Figure 7. Width and amplitude of solitary wave structure in the absence of dust $n_{e0} = 0.50 \times 10^9 \text{cm}^{-3}$ and positive ion density $n_{+0} = 1 \times 10^9 \text{cm}^{-3}$ negative

ion density $n_{-0} = 0.50 \times 10^9 \mathrm{cm}^{-3}$ with $T_e = 1.1604 \times 10^5 \mathrm{K}$, $T_+ = 1.1604 \times 10^4 \mathrm{K}$, $T_- = 1.1604 \times 10^4 \mathrm{K}$, $l_x = 0.80$, $B_0 = 5$ Gauss and $u_0 = 0.06$ a (fast mode) b (slow mode).

5. Conclusion

We have investigated the nonlinear electrostatic solitary wave structures in negative ion magnetized plasma in the presence of stationary dust. Both type of ions are adiabatically heated and electron follow Boltzmann distribution. It is seen that compressive and refractive solitons are observed for fast and slow modes in negative ion magnetized plasma in the presence of stationary dust. It is noted that decrease in the magnetic field enhance the width of solitary wave structure. It is also observed that variation in the temperatures of both type of ions effect the profile of non linear solitary structures. It is noted that in the presence of dust the amplitude and width of the non linear solitary structure modifies significantly. Our findings have relevance with laboratory and space plasmas where positive and negative ions are present with stationary dust.

Appendix

$$-\lambda \partial_X n_+^{\ 2} + l_x \partial_X u_{+x}^{\ 2} = f_1 \tag{26}$$

$$-\lambda \partial_X u_{+x}^{\ 2} + l_x \partial_X \Phi^{\ 2} + \frac{5\alpha l_x}{3} \partial_X n_+^{\ 2} = f_2 \tag{27}$$

$$l_y \partial_Y \Phi^{\ 2} - \Omega u_{+z}^{\ 2} + \frac{5\alpha l_y}{3} \partial_Y n_+^{\ 2} = f_3 \tag{28}$$

where $f_1 = -\partial_\tau n_+^{\ 1} - l_x \partial_X \ n_+^{\ 1} u_{+x}^{\ 1} \ - l_y \partial_Y \ u_{+y}^{\ 1}$,

$f_2 = -\partial_\tau u_{+x}^{\ 1} - l_x u_{+x}^{\ 1} \partial_X u_{+x}^{\ 1} + \frac{5}{9} \alpha l_x n_+^{\ 1} \partial_X n_+^{\ 1}$, and

$f_3 = \lambda \partial_X u_{+y}^{\ 1} + \frac{5}{9} \alpha l_y n_+^{\ 1} \partial_Y n_+^{\ 1}$ have been defined.

Now collecting the next higher order terms i.e., ($\varepsilon^{5/2}$) from continuity and the momentum equation of negative ions, we have

$$-\lambda \partial_X n_-^{\ 2} + l_x \partial_X u_{-x}^{\ 2} = f_4 \tag{29}$$

$$-\lambda \partial_X u_{-x}^{\ 2} - \delta l_x \partial_X \Phi^{\ 2} + \frac{5\beta\delta l_x}{3} \partial_X n_-^{\ 2} = f_5 \tag{30}$$

$$-\delta l_y \partial_Y \Phi^{\ 2} + \delta\Omega u_{-z}^{\ 2} + \frac{5\beta\delta l_y}{3} \partial_Y n_-^{\ 2} = f_6 \tag{31}$$

where $f_4 = -\partial_\tau n_-^{\ 1} - l_x \partial_X \ n_-^{\ 1} u_{-x}^{\ 1} \ - l_y \partial_Y \ u_{-y}^{\ 1}$,

$f_5 = -\partial_\tau u_{-x}^{\ 1} - l_x u_{-x}^{\ 1} \partial_X u_{-x}^{\ 1} + \frac{5}{9} \beta\delta l_x n_-^{\ 1} \partial_X n_-^{\ 1}$, and

$f_6 = \lambda \partial_X u_{-y}^{\ 1} + \frac{5}{9} \beta\delta l_y n_-^{\ 1} \partial_Y n_-^{\ 1}$

have been defined.

The next higher order (ε^{2} 2) term for electron and Poisson equations give,

$$n_e^{\ 2} = \Phi^{\ 2} + \frac{1}{2} \ \Phi^{\ 1 \ \ 2} \tag{32}$$

$$n_+^{\ 2} - P n_-^{\ 2} - Q n_e^{\ 2} = f_7 \tag{33}$$

where

$$f_7 = l_x^{\ 2} \partial_X^{\ 2} \Phi^{\ 1} + l_y^{\ 2} \partial_Y^{\ 2} \Phi^{\ 1} + \frac{Q}{2} \ \Phi^{\ 1 \ \ 2}$$

has been defined. On solving Eqs.(26-33) and using the first order expressions of perturbed quantities, we find the relation after some simplification as follows,

$$\begin{aligned} &\left| 3\lambda \ 5\beta\delta l_x^{\ 2} - 3\lambda^2 \ f_1 + 3 l_x^{\ 2} \ 5\beta\delta l_x^{\ 2} - 3\lambda^2 \ f_2 - 3P\lambda \ 5\alpha l_x^{\ 2} - 3\lambda^2 \ f_4 \right| \\ &- \left| 3P l_x \ 5\alpha l_x^{\ 2} - 3\lambda^2 \ f_5 - \ 5\alpha l_x^{\ 2} - 3\lambda^2 \ \ 5\beta\delta l_x^{\ 2} - 3\lambda^2 \ \partial_X f_7 = 0 \right| \end{aligned} \tag{34}$$

where Eq.(23) has been used to obtain the above relation.

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