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FORMATION OF LOPSIDED AND BAR STRUCTURES IN NON-STATIONARY GRAVITATING SYSTEMS: II-ANISOTROPIC CASE

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In the present study, growth rates of non-stationary model of radially gravitating disk with an anisotropic velocity diagram is constructed. We have focused on two oscillation modes i.e. lopsided and bar-like modes. Critical diagrams of the initial virial ratio and rotation parameter are obtained for these two perturbation oscillation modes. A comparative analysis shows that lopsided mode again dominate the bar-like mode, like its isotropic case, against all the values of rotation parameter for anisotropic non-stationary model.

Keywords : Pulsation, Non-stationary, Anisotropic model, Potential perturbation, Virial ratio, Critical value

1. Introduction

In the recent years unprecedented progress in observational Cosmology has revealed a great deal of information about the formation and evolution of structures in this beautiful star studded Universe [1,2,4]. This, in turn, has raised many challenging questions for researchers. The images obtained by Hubble Space Telescope reveals that basic large scale structures are shaped at non-stationary, non-linear stage of their evolution; therefore, modern extragalactic astronomy is compelled to study early non-linear stages of evolution of self-gravitating systems. Several specific stationary phase models of gravitating systems have been worked out by numerous authors. A large number of results have been obtained in Binney and Tremaine (1987). Nuritdinov (1991) constructed non-stationary generalization of Zeldovich's pancakes.

In part I of this paper, we mentioned the gravitational instability of lopsided and bar-like perturbation modes by using Nuritdinov's non-stationary isotropic model. In this paper, we examine the gravitational instability for non-stationary anisotropic model for the same modes.

We again consider the following initial state of non-linearly non-stationary phase model for a collision-less self-gravitating disk, which we described [1].

$$\Psi(r,v_r,v_\perp,t)=\frac{\sigma_o}{2\pi\Pi\sqrt{1-\Omega^2}}\left[\frac{1-\Omega^2}{\Pi^2}\left(1-\frac{r^2}{\Pi^2}\right)-(v_r-v_a)^2-(v_\perp-b_b)^2\right]^{-\frac{1}{2}}\chi(R-r) \qquad (1)$$

which undergoes radial pulsations of the form $R=R_o\Pi(t)$, where

$$\Pi(t)=\frac{1+\lambda\cos\psi}{1-\lambda^2} \qquad (2)$$

$$t=\frac{\psi+\lambda\sin\psi}{(1-\lambda^2)^{3/2}} \qquad (3)$$

and the amplitude $\lambda=1-\left(\frac{2T}{|U|}\right)_o$ of the radial oscillations is given in terms of a virial relation at time t=0, i.e., $0\le\lambda\le1$. In equation (1) we have chosen the normalization $\pi^2G\sigma_o=2R_o$, (R_o=1), while Ω is a dimension-less parameter characterizing the magnitude of the rigid-body rotation and χ is the Heaviside function. The other parameters are already described in detail [1]. In the following we shall also need an expression, which follows from equation (1), for the surface density in the initial state.

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$$\sigma(\vec{r},t) = \frac{\sigma_o}{\Pi^2(t)}\sqrt{1-\frac{r^2}{\Pi^2(t)}} \tag{4}$$

The period of the model radial pulsation is $P(\lambda) = 2\pi(1-\lambda^2)^{-3/2}$.

2. Formation of Anisotropic Model

We noted that non-linearly non-stationary model (1) has an isotropic velocity diagram. Since this is a somewhat idealized picture, the question arises of also examining at least one anisotropic model with a radial non-stationary, in initial state. To do this, it is appropriate to average over the parameter Ω in the standard way,

$$\Psi_a = \frac{\int_{-1}^{1}\rho(\Omega)\,\Psi d\Omega}{\int_{-1}^{1}\rho(\Omega)\,d\Omega} \tag{5}$$

Nuritdinov [2] constructed a rotating anisotropic model that examines a weighting function in equation (5) of the form

$$\rho(\Omega') \frac{2}{\pi}\sqrt{1-\Omega'^2}(1+\Omega\Omega') \tag{6}$$

Equation (4) and equation (5), by the method described [11], implies that

$$\psi_a = \frac{\sigma_o}{\pi}\left[1+\Omega(xv_y - yv_x)\right]\chi(B) \tag{7}$$

where Ω again plays the role of rotation parameter, while

$$B = \left(1-\frac{r^2}{\Pi^2}\right)(1-\Pi^2 v_\perp^2) - \Pi^2(v_r - v_a)^2 \tag{8}$$

with

$$V_a = -\lambda\sqrt{1-\lambda^2}\,\frac{r\sin\psi}{\Pi^2} \tag{9}$$

The model (6) is convenient in that, with a non-stationary analog of the dispersion relation for an isotropic model (1) it is easy to derive this kind of equation for an anisotropic model (6) without having to do the needed calculations repeatedly. For this it is sufficient to apply the procedure (5) to the non-stationary analog of the dispersion relation for an isotropic mode.

3. Non-Stationary Dispersion Relation (NADR)

Using equation (5) on non-stationary dispersion relation of isotropic model (1) for lopsided mode [1], the following non-stationary dispersion relation for anisotropic model is obtained [12].

$$\Lambda r_k(\psi) = \frac{3(\lambda+\cos\psi)^{3-k}(\sin\psi)^{k-1}}{4(1+\lambda\cos\psi)}F(\psi)\;;\; k = 1,2,3 \tag{10}$$

where

$$\begin{aligned} F(\psi) = {} & \left\{\begin{array}{l} 11\left[4(\lambda+\cos\psi)^2 - (1-\lambda^2)\sin^2\psi\right] - \\ 10i\Omega\sqrt{1-\lambda^2}(\lambda+\cos\psi)\sin\psi \end{array}\right\} M_o(\psi) \\ & 10\left\{\begin{array}{l} 11(1-\lambda^2)(\lambda+\cos\psi)\sin\psi + \\ i\Omega\sqrt{1-\lambda^2} \\ \left[(\lambda+\cos\psi)^2 - (1-\lambda^2)\sin^2\psi\right] \end{array}\right\} M_1(\psi) \\ & \left\{\begin{array}{l} 11(1-\lambda^2)\left[\begin{array}{l} 4(1-\lambda^2)\sin^2\psi - \\ (\lambda+\cos\psi)^2 \end{array}\right] \\ +10i\Omega(1-\lambda^2)^{3/2}(\lambda+\cos\psi)\sin\psi \end{array}\right\} M_2(\psi) \end{aligned} \tag{11}$$

and Λ is already describe [1].

While, on applying equation (4) on non-stationary dispersion relation of isotropic model for bar-like mode as discussed [1], the same equation has been achieved without any change.

$$\Lambda L_\tau(\psi) = \frac{3(\lambda+\cos\psi)^{1-\tau}(\sin\psi)^{\tau}}{2(1+\lambda\cos\psi)^2}B(\psi) \quad ;\; \tau = 0-1 \tag{12}$$

where

$$B(\psi) = \left(\cos\psi + \lambda - i\Omega\sin\psi\sqrt{1-\lambda^2}\right)L_o(\psi) + QL_1(\psi) \tag{13}$$

Using the formula of growth rates mentioned in [4,8,9] and these two non-stationary dispersion equations (10) & (12), we can get the graphs of critical dependence of virial ratio on rotation parameter (Figures 1 and 2).

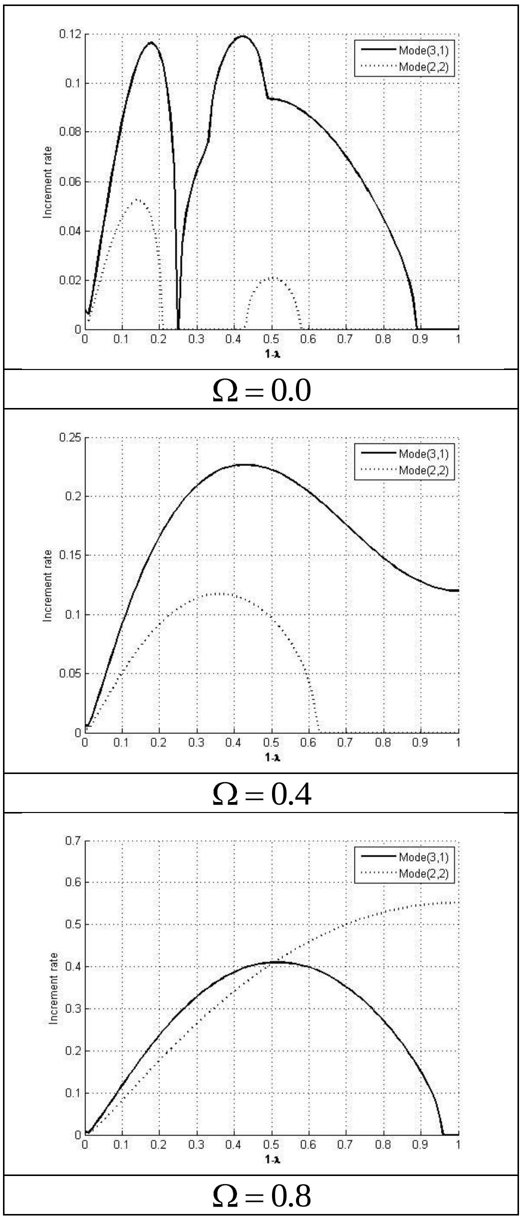

Figure 1 The growth rates instabilities of disk like self-gravitating system in which lopsided structures has highest growth rates (increment) than bar-like structures against the background of non-stationary anisotropic model at $\Omega =, 0.0, 0.4, 0.8$.

4. Results and Discussion

The above study reveals that non-stationary analogs of the dispersion relation for lopsided and bar-like modes with the background of radially pulsating self-gravitating non-stationary disk-like models. Using growth rates formula, we obtained dependence of virial ratio on rotation parameter Ω for both oscillation modes, in the form of graphs. Mostly at all values of rotation parameter, lopsided structures has higher growth rates than the bar-like structures, except at $\Omega = 0.8$ with virial ratio > 0.5, $\Omega = 0.6$ with virialratio > 0.55, at $\Omega = 0.2$ with $1-\lambda > 0.3$ and $\Omega = 0.8$ with virial ratio > 0.5. For $\Omega = 0.0, 0.4, 1.0$, lopsided structures always more unstable than the bar-like structures.

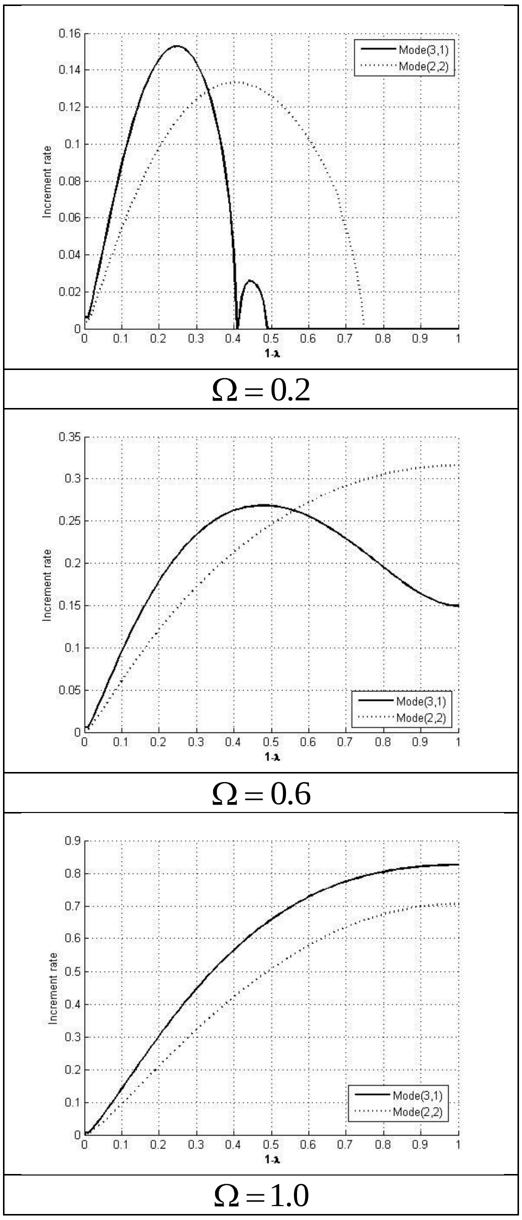

Figure 2. The growth rates instabilities of disk like self-gravitating system in which lopsided structures has highest growth rates (increment) than bar-like structures against the background of non-stationary anisotropic model at $\Omega =, 0.2, 0.6, 1.0$.

5. Conclusion

Graphical patterns clearly shows that lopsided structures has again clear advantage over bar-like mode for all values of rotation parameter same as in the case of isotropic model, therefore, it is concluded that the formation of lopsided structures has greater probability than bar-like structures in the early evolution of the Universe.

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