

## A Two Step Empirical Method for the Determination of Effective Thermal Conductivity of Multi-Phase Porous Solids

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A two step empirical approach is proposed to estimate the effective thermal conductivity of porous solids. The total thermal conductivity of solid phase is calculated by assuming different minerals arranged in "parallel". Then the final effective thermal conductivity is calculated by taking in to account the porosity content and an additional empirical parameter related to geometry of pore. It is shown that the effective thermal conductivity of a porous rock can be successfully modeled from the thermal conductivity of constituent mineral phases determined by x-ray diffraction and porosity measurements by standard methods.

**Keywords:** Effective thermal conductivity, Porosity, Heat flow, Bounds on conductivity, Modeling

### 1. Introduction

The effective thermal conductivity ( $\lambda_e$ ) of a solid material composed of several phases is mostly required to be known beforehand in applied science. This quantity is related to volume concentration and thermal conductivities of the constituent components, and their structure and distribution in the sample. Information about thermal properties of porous materials such as rocks is needed for calculations of heat dissipation from underground nuclear explosions and for the rate of heat losses from earth due to geothermal gradients [1]. Modeling of thermal properties has remained of much interest to researchers for various material design and industrial applications *e.g.* for thermal barrier coatings [2], nano-particles suspensions [3], porous materials [4-6], soils [7], and graphene-based films [8]. Thermal conductivity of soil and rock aggregates alongwith the insulation material is a necessary parameter for determination of current carrying capacity of buried cables and of heat losses from underground steam and hot water pipes.

In connection with the oil recovery processes, the properties of interest are thermal conductivity ( $\lambda$ ), the thermal diffusivity ( $\kappa$ ), and specific heat  $C_p$ . Since these three quantities are interrelated, ( $\kappa = \lambda / \rho C_p$ ), where  $\rho$  is density), from known values of any two of these parameters third one can be obtained easily. Laboratory measurement techniques of thermal conductivity are time consuming in addition of specific sample preparation requirements and equipment. Various theoretical models have been proposed to estimate the effective thermal conductivity of a composite from the components' conductivity and material's structure. Most of which are empirical and few are based on the

effective medium theory. This work presents empirical formulation for the estimation of thermal conductivity of a multiphase solid composite with minor porous phase.

### 2. Modeling Effective Thermal Conductivity

In the most general sense, a porous solid can be thought of a heterogeneous mixture of numerous solid phases and a fluid phase (gas/air/oil etc.) with their own intrinsic thermal conductivities and structural distribution. Starting from a very simple case of a porous solid consisting of solid phase with effective thermal conductivity,  $\lambda_s$ , and pores with porosity,  $\phi$ , filled with a relatively low conductivity fluid of thermal conductivity,  $\lambda_f$ , the overall thermal conductivity for such a system will be expected to lie somewhere between the conductivities of two phases (solid and fluid) because  $\lambda_f < \lambda_s$  for situations considered here. It is distribution of these two quantities in the material that determines effective conductivity of that material under the limit that no convection is possible. As far as existing models to estimate thermal conductivity are concerned, the simplest case is of a two phase solid, the weighted averages of the conductivities that correspond to the well known "parallel" and "series" models [1], respectively, yield the following effective thermal conductivity ( $\lambda_e$ ):

For Series model (Figure 1a)

$$\lambda_e = \phi \lambda_f + (1 - \phi) \lambda_s \quad (1a)$$

For Parallel model (Figure 1b)

$$1 / \lambda_e = \phi / \lambda_f + (1 - \phi) / \lambda_s \quad (1b)$$

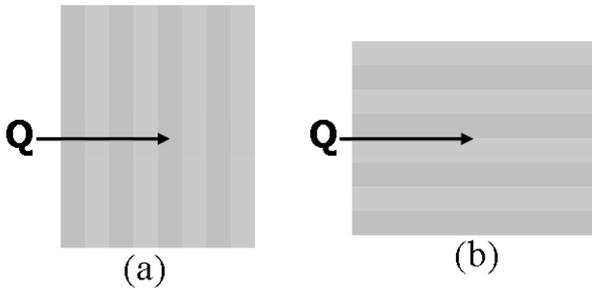


Figure 1. (a) Series and (b) parallel; combination (with respect to heat flow  $Q$ ) of two phase material with thermal conductivities,  $\lambda_s$  and  $\lambda_f$ .

In the “series” distribution, the two phases are assumed to be thermally in series with respect to direction of heat flow ( $Q$ ), whereas in “parallel” distribution, the two phases are in parallel with respect to direction of heat flow. This simple model is illustrated in Figure 1 (a, b), which depicts that “series” model has more thermal resistance (minimum conducting) while “parallel” model has less thermal resistance (maximum conducting). These two approximations represent upper and lower bounds on the effective conductivity of a two phase solid material as shown in Figure 4 and are extremely limiting cases hardly to be applicable to a real situation. If the material is supposed to be consisting of two solid phases with conductivities  $\lambda_1$  and  $\lambda_2$  with volume fractions  $v_1$  and  $v_2$ , respectively, then the above equations would be modified accordingly. The same principle is applicable to mathematical relations given in the whole text.

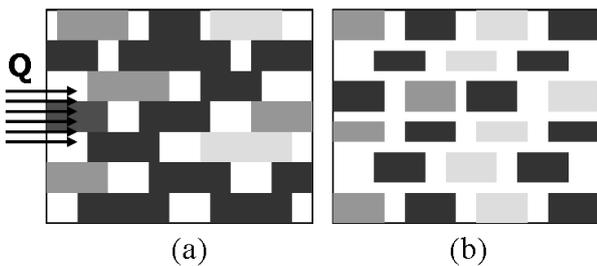


Figure 2. Two possible arrangements of regularly shaped objects: (a) fluid areas (white) are not interconnected while solid areas are interconnected, (b) fluid phase continuous while solid phase is distributed randomly.

Figure 2 shows somewhat improved distribution of such a dispersed media and corresponds to the following relations [9] :

For Figure 2a

$$\lambda_e = 4\lambda_s \left( \frac{\varphi}{1 + \lambda_s / \lambda_f} \right) + \lambda_f (1 - 2\varphi) \quad (2a)$$

For Figure 2b

$$\lambda_e = 4\lambda_s \left\{ \left( \frac{1 - \varphi}{1 + \lambda_s / \lambda_f} \right) \right\} + 2\lambda_s (\varphi - 1) \quad (2b)$$

The above relations are modified further when (i) solid phase acts as a minor phase and uniformly distributed in fluid as a major phase, (ii) fluid phase is a minor phase uniformly distributed in solid phase. This situation is shown in Figure 3 and the effective thermal conductivity can be expressed as [9] :

For Figure 3a

$$\lambda_e = \lambda_f \frac{[\varphi^{2/3} + \{(\lambda_f / \lambda_s)(1 - \varphi^{2/3})\}]}{[(\varphi^{2/3} - \varphi) + \{(\lambda_f / \lambda_s)(1 - \varphi^{2/3} + \varphi)\}]} \quad (3a)$$

For Figure 3b

$$\lambda_e = \frac{[\lambda_s (1 - \varphi)^{2/3} + (\lambda_s / \lambda_f) \{1 - (1 - \varphi)^{2/3}\}]}{(-\varphi)^{2/3} - (1 - \varphi) + (\lambda_s / \lambda_f) \{(2 - \varphi) - (1 - \varphi)^{2/3}\}} \quad (3b)$$

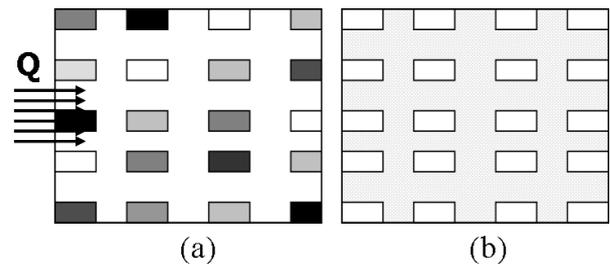


Figure 3. Regular distribution of solid (shaded blocks) and fluid (white background) phases: (a) fluid as a major phase and solid as a minor phase, (b) solid as a major phase and fluid as a minor phase.

The provisional character of equations 3a and 3b is due to the fact that an actual material cannot be thought of as consisting of particles not having contact with each other throughout the solid phase. More realistic approach may be like that of cubic and tetrahedral arrangement of grains of a dispersed system presupposing spheres of equal sizes and no dependence of the degree of filling of spaces on the size of the constituent phases.

Bounds on conductivity proposed by Hashin and Shtrikman [10] are tighter than series and parallel models (see Figure 4 for more clarity) but are still far away in terms of ratios of fluid to solid thermal conductivities ( $\lambda_f / \lambda_s < 0.01$ ) and may be expressed as (Figure 4):

$$X_1 \leq \lambda_e \leq X_2 \quad (4)$$

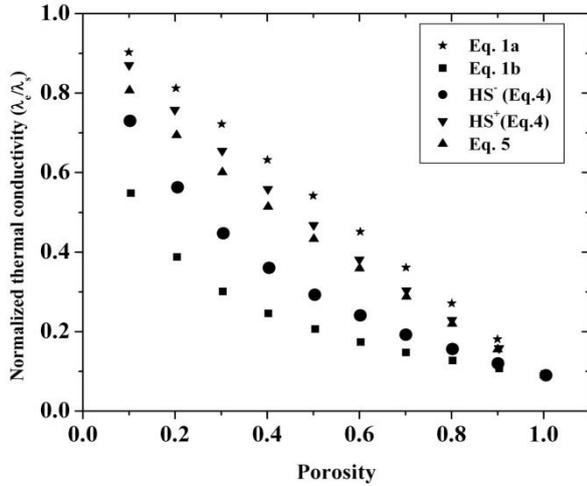


Figure 4. Porosity dependence of normalized thermal conductivity ( $\lambda_e/\lambda_s$ ) for  $\lambda_f/\lambda_s = 0.1$  as described by models in equations 1, 4 and 5.

where

$$X_1 = \lambda_f + \frac{[3\lambda_f(\lambda_s - \lambda_f)(1 - \phi)]}{[3\lambda_f + \phi(\lambda_s - \lambda_f)]} \quad \text{and}$$

$$X_2 = \lambda_s + \frac{\{3\lambda_3(\lambda_f - \lambda_s)\phi\}}{\{3\lambda_s + (1 - \phi)(\lambda_f - \lambda_s)\}}$$

Another empirical relation given by Ribaud [11] lies in between Hashin-Shtrikman bounds and is shown in Figure 4:

$$\lambda_e = \lambda_f \phi^{1/3} + (1 - \phi)^{2/3} \quad (5)$$

According to Brailsford and Major [9] the effective thermal conductivity of a two phase material (with conductivities  $\lambda_1$  and  $\lambda_2$ , and  $v_1$  as volume fraction of phase 1) by assuming phase 2 as continuous phase could be approximated by :

$$\lambda_e = \lambda_2 \left( \frac{A - 2v_1 B}{A + v_1 B} \right) \quad (6)$$

where;  $A = 2 \frac{\lambda_2}{\lambda_1} + 1$  and  $B = \frac{\lambda_2}{\lambda_1} - 1$

This model is similar to that proposed by Maxwell [12] of randomly distributed solid phase in a continuous fluid phase, and can be extended to a mixture of spherical particles of either phase distributed in the second phase.

The estimation of effective thermal conductivity of a porous rock, consisting of a solid phase (major/continuous phase) and a fluid phase (minor phase), seems much complicated because it is essentially a heterogeneous mixture of numerous solid

and fluid components with their own intrinsic thermal conductivities. The solid phase is further a random mixture of multiple sub-component minerals of very different compositions and completely random structural distributions and how to assign a proper value of  $\lambda_s$  to solid phase. The minor fluid phase on the other hand may be assumed as a single phase (either porosity content or air/gas/oil etc.) to a good approximation with well established values of  $\lambda_f$ . To avoid time consuming and cumbersome sample preparation methods and complicated experimental techniques for the determination of effective thermal conductivity of a multiphase heterogeneous system, a two step approach is proposed in this article. Firstly, the thermal conductivity of solid phase is computed by using thermal conductivities of individual mineral components and their respective volume fractions. Secondly, net effective thermal conductivity is calculated by applying porosity correction by a suitable formulation. The mineral contents can be determined by taking x-ray diffraction data of a representative sample of rock whereas porosity values can be obtained by standard density measurements. Let us consider the solid as compact with no porosity (low conductivity phase) and suppose that the constituent phases with conductivity,  $\lambda_i$ , and volume fractions  $V_i$ , are arranged in parallel (resulting in maximum conductivity), then effective conductivity of the matrix will be:

$$\lambda_s = \sum \lambda_i V_i \quad (7)$$

The effective thermal conductivity of a porous rock may be then calculated by the following relationship [13] ;

$$\lambda_e = (1 - \Delta)\lambda_s + \Delta\lambda_f \quad (8)$$

where

$$\Delta = \left\{ \frac{2n}{2n-1} \right\} \left\{ \frac{1}{1-\phi^n} \right\}$$

$n > 0$  is an empirical exponent depending on the porosity, shape, and orientation of the pore. This scheme essentially is a parallel-parallel model in a porous rock where the solid phase acts as continuous phase (interconnected) and the matrix should not look like open pore structure type. Normalized thermal conductivity is plotted in Figure 5 for different  $n$  values. The proposed scheme of estimating thermal conductivity of porous rocks was applied to the published data on porous sandstone by Woodside and Messmer [1] where mineral and porosity contents are also known. A good agreement is found between measured and estimated effective thermal conductivities as shown in Figure 6. It can be seen that for certain limit

of porosity, the conductivity is overestimated while below this limit the reverse is true. The plot in Figure 6 suggests that for highly porous medium, this model seems to have certain limitations.

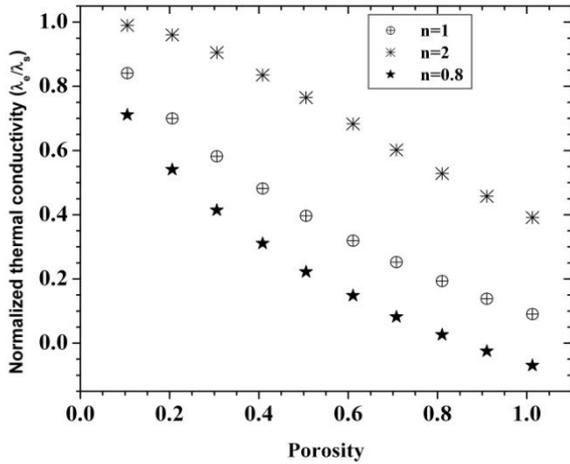


Figure 5. Plot of normalized thermal conductivity ( $\lambda_e/\lambda_s$ ) as a function of porosity as described by model in equation 8 for different values of exponent  $n$ .

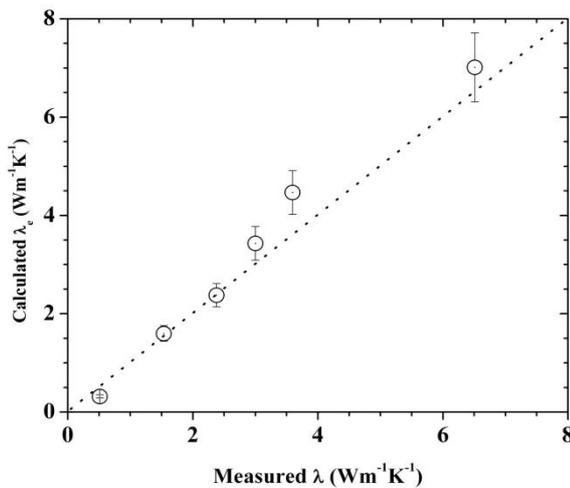


Figure 6. Measured (Woodside and Messmer, measured at a temperature of 303 K) and calculated thermal conductivities of sandstone for  $n=7$  in equation 8.

### 3. Summary

In summary, various theoretical models for the estimation of effective thermal conductivity of porous solids has been reviewed and discussed. To predict the effective thermal conductivity of multiphase porous rocks based on mineral and porosity contents, a two step empirical model is proposed in this work. The total thermal conductivity of solid phase is calculated by assuming different minerals arranged in “parallel”. Then the final effective thermal conductivity is calculated by taking in to account the porosity content and an additional empirical parameter related to geometry of pore. The proposed scheme has shown good agreement between measured and calculated conductivities; however has a limitation for very high porosity contents.

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