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Picture Fuzzy Linguistic Sets and Their Applications for Multi-Attribute Group Decision Making Problems

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ABSTRACT

Having encouraged by the linguistic term in decision models, it is proposed a method of multi attribute group decision making. This amalgamates the idea of picture fuzzy sets and linguistic term sets to discourse the situations where the real-life problems fail to express in numerical form. Firstly, it is introduced the concept of picture fuzzy linguistic number and comparison rules for ranking the alternatives are discussed. Further the aggregation operators based on picture fuzzy linguistic information are introduced. Finally, it is introduced a technique to obtain satisfactory results about real-life complex problems, and it is given a descriptive example to discuss the reliability and effectiveness of the suggested technique by using group decision criteria.

1. Introduction

In recent years, Picture fuzzy sets (PFSs) have attained much attention of researchers and are extensively spotlighted on the theory of decisions. In decision theory, multi attribute group decision making (MAGDM) method is one of the best techniques to utilize to rank the alternatives or to choose the best one option from concerned criteria. However, there are some cases which are unmanageable for researchers to express the preference in doing MAGDM problems due to uncertainties, imprecise and inexact information. Based on these circumstances fuzzy sets (FSs), developed by Zadeh [1] were initially used. In FSs each of the elements contained only one index namely as degree of membership $"P(x)"$ introduced by utilizing the crisp values, which oscillate from 0 and 1. Non-membership degree for the FS is straightforward equivalent to $"1-P(x)"$. However, sometime FSs fail to handle such cases where membership degree contains uncertainties or inexact information. In such condition, it is difficult to define on crisps values. However, interval-valued fuzzy sets (IVFSs) developed by Zadeh [2], to apprehend the uncertainties or inexact information about degree of membership.

Sometime FS has some drawbacks for example, it has no ability to show the neutral state (which neither favor nor disfavor).Based on these circumstances, intuitionistic fuzzy sets (IFSs) developed by Atanassov [3], were initially used. In IFSs each of the elements contained two indices namely as degree of membership $"P(x)"$ and degree of non-membership $"N(x)"$ with condition that $0 \le P(x)+N(x) \le 1.$ Degree of neutral membership for the IFSs will be calculated straightforward equivalent to $1-(P(x)+N(x)).$ Some applications related to IFSs have been discussed previously [4-7].

Later, the degree of membership and non-membership in IFSs may be denoted as interval values alternatively by crisp numbers. So, to the interval valued intuitionistic fuzzy sets (IVIFSs), developed by Atanassov and Gargov [8]. IVIFSs are an extension of FSs & IFSs. After that some problems arise that to find out the neutral membership independently. Then IVIFSs [9] fails to capture any satisfaction about the independency of neutral membership. Based on these circumstances, the idea of picture fuzzy sets (PFSs), developed by Cuong [10] were initially used. In PFSs each of the elements contained three indices namely as degree of membership $"P(x)"$, degree of non-membership $"N(x)"$ and degree of neutral membership $"I(x)"$ with condition that

$$0 \le P(x)+I(x)+N(x) \le 1.$$

In 2018, Ashraf et al. [11] introduced some methods to deal with PFNs. In multiple attribute group decision making (MAGDM), people usually evaluate each index in a natural language with some linguistic information. Effective linguistic group decision making is critical to the efficient quantitative expression of the language information in MAGDM, as well as to effective aggregation models and their algorithms. Zadeh [1, 2] introduced and developed the theory of approximate reasoning based on the notions of the linguistic variable and the fuzzy set to deal with uncertain

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decision environments. Finding a proper way to aggregate the preferences of decision makers is of significance to decision making. Some applications related to linguistic models are discussed elsewhere [12, 13].

The objectives of this paper are: (1) to introduce the picture fuzzy linguistic sets (PFLSs), (2) to define the picture fuzzy linguistic numbers (PFLNs) and related basic operational identities, (3) to suggest score, accuracy, certainty functions for comparison, (4) to propose the PFLN weighted arithmetic average (PFLNWAA), the PFLN weighted geometric average (PFLNWGA) operators and to investigate related properties, and (5) to demonstrate a MAGDM method based on the PFLNWAA and PFLNWGA operators under picture fuzzy linguistic information.

The superfluity of this paper is planned as follows. Section "Preliminaries" gives brief reassess the initial ideas related to LTSs, PFSs and their properties. Next section "Picture Fuzzy Linguistic sets and their Operations" gives complete details about PFLSs, PFLNs and their operational properties. The next sections "Comparison Rules for PFLNs" and "Weighted Aggregated Operators for PFLNs" defines a rule which utilized to rank the alternatives. In sections "MAGDM method utilizing the PFLNWAA and PFLNWGA operators", attribute MAGDM method is proposed to deal with picture fuzzy linguistic information and in the end a descriptive example is illustrated to express the effectiveness and reliability of the suggested technique. Finally, a conclusion and references are given.

2. Preliminaries

In this article, we give a brief discussion on some basic concepts and definitions related to PFSs combining the concept of linguistic term sets and some more familiarized concepts which are utilized in following analysis.

2.1 Definition [14]

Suppose that $Ų=\{u_0, u_1, u_2, .. u_{t-1}\}$ is a finitely ordered discrete linguistic term set (LS), where t is the odd cardinality with $t>0$. Where u_i represents the linguistic values (LVs) of the linguistic term set Ų.

Then any two LVs u_j, u_k of the LS Ų must satisfy the below characteristics.

1. Ordering in a set; $Ų_j \geq Ų_k$ if $j \geq k$,
2. Negation Operator; $neg(u_k) = u_{t-1-k}$,
3. Max. Operator; $\max(u_j, u_k) = u_j$ if $j > k$,
4. Min. Operator; $\min(u_j, u_k) = u_k$ if $j > k$.

Extension in discrete linguistic term set is continuous linguistic term set which described as

$$Ų = \{u_j \mid j \in R^+\}$$

The benefit of continuous linguistic term set is that its preserve the strictly monotonically increasing condition [3, 8].

2.2 Definition [15]

For any two LVs u_j, u_k of the LS Ų preserve some operational properties.

1. $\tau u_k = u_{\tau \times k}, \tau \geq 0$
2. $u_j + u_k = u_{j+k}$
3. $u_j \times u_k = u_{j \times k}$
4. $(u_j)^\tau = u_{j^\tau}$.

2.3 Example

Suppose that

$$Ų = \{u_0, u_1, u_2, u_3, u_4\} = \left\{\begin{matrix} verypoor, poor, \\ medium, rich, veryrich \end{matrix}\right\}$$

is a finitely ordered discrete linguistic term set and we choose any two-linguistic term say u_2 and $u_3 \in Ų$, then by Definition 2.2 operational properties are presented as

$$0.4u_2 + 0.6u_3 = u_{0.4\times2} + u_{0.6\times3} = u_{0.8} + u_{1.8} = u_{0.8+1.8} = u_{2.6}$$

Since the attained outcome is not lie in proper linguistic terms, the result lies between u_2 ("*medium*") and u_3 ("*rich*") but its better approximate to u_3.

2.4 Definition [16]

Let R be a universe set and then an PFS A in R is defined as:

$$A = \{\langle r, P_A(r), I_A(r), N_A(r) \mid r \in R\rangle\},$$

where $P_A: R \to [0,1]$, $I_A: R \to [0,1]$ and $N_A: R \to [0,1]$ are said to be degree of positive-membership of r in R, neutral-membership degree of r in R and negative-membership degree of r in R respectively. Also P_A, I_A *and* N_A satisfy the following condition,

$$(\forall r \in R)\ (0 \leq P_A(r) + I_A(r) + N_A(r) \leq 1).$$

3. Picture Fuzzy Linguistic sets and their Operations

The concept of linguistic variable was introduced by Zadeh [1, 2]. A linguistic value is less precise than a crisp number, but it is closer to human cognitive processes that are used to solve uncertainty problems successfully.

Picture fuzzy linguistic variables provide the degrees of positive membership, neutral membership and negative membership which is more-or-less independent from each other, and the only requirement is that the sum of these three degrees is not greater than 1. Picture fuzzy linguistic variables are higher order intuitionistic fuzzy linguistic variables. The application of higher order fuzzy linguistic variables makes the solution procedure more complex, but if the complexity of the computation time, computation volume, or memory space is not the matter of concern, then a better result can be achieved. In this environment, individual opinions are represented by picture linguistic preference relations.

3.1 Definition

Suppose that R be a universe set with generic point (object) $r \in R$ and $Ų = \{u_0, u_1, u_2, ..u_{t-1}\}$ be a finitely ordered discrete linguistic term set, where t is the odd cardinality with $t > 0$. A PFLS A in R presented as

$$A = \{\langle r, u_{\theta(r)}, (P_A(r), I_A(r), N_A(r)) \mid r \in R\rangle\},$$

where $u_{\theta(r)} \in Ų$, $P_A: R \to [0,1]$, $I_A: R \to [0,1]$ and $N_A: R \to [0,1]$ are said to be degree of positive-membership of r in R, neutral-membership degree of r in R and negative-membership degree of r in R respectively. Also P_A, I_A *and* N_A satisfy the following condition:

$$(\forall r \in R)\ (0 \le P_A(r) + I_A(r) + N_A(r) \le 1).$$

For PFLS $\{\langle r, u_{\theta(r)}, (P_A(r), I_A(r), N_A(r)) \mid r \in R\rangle\}$, which are quadruple components

$$\langle u_{\theta(r)}, (P_A(r), I_A(r), N_A(r))\rangle$$

are said to PFLN and each PFLN can be denoted by $e = \langle u_e, (P_e, I_e, N_e)\rangle$, where $u_e \in Ų$, P_e, I_e and $N_e \in [0,1]$, with condition that

$$0 \le P_e + I_e + N_e \le 1$$

Therefore, when $P_e = 1$ and $I_e = N_e = 0$ the PFLN tern into the linguistic term.

3.2 Definition

Let $e_j = \langle u_{e_j}, (P_{e_j}, I_{e_j}, N_{e_j})\rangle$ and $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k})\rangle$ be any two PFLNs and $\tau \ge 0$. Then the operations of PFLNs can be denotes as

1. $\tau e_j = \left\langle u_{\tau \times e_j}, \left(1-(1-P_{e_j})^\tau,\ (I_{e_j})^\tau,\ (N_{e_j})^\tau\right)\right\rangle;$
2. $e_j + e_k = \langle u_{e_j} + u_{e_k}, (P_{e_j} + P_{e_k} - P_{e_j} \cdot P_{e_k},\ I_{e_j} \cdot I_{e_k},\ N_{e_j} \cdot N_{e_k})\rangle;$
3. $e_j \times e_k = \langle u_{e_j} \times u_{e_k}, (P_{e_j} \cdot P_{e_k},\ I_{e_j} \cdot I_{e_k},\ N_{e_j} + N_{e_k} - N_{e_j} \cdot N_{e_k})\rangle;$
4. $e_j^\tau = \left\langle (u_{e_j})^\tau, \left((P_{e_j})^\tau,\ (I_{e_j})^\tau,\ 1-(1-N_{e_j})^\tau\right)\right\rangle.$

3.3 Example

If $Ų = \{u_0, u_1, u_2, u_3, u_4\}$, $e_1 = \langle u_1, (0.6, 0.3, 0.1)\rangle$, $e_2 = \langle u_2, (0.4, 0.3, 0.3)\rangle$ and $\tau = 2$. Then by Definition 3.2 following outcomes can be obtained as

1. $2e_1 = \left\langle u_{2\times 1}, \left(1-(1-0.6)^2,\ (0.3)^2,\ (0.1)^2\right)\right\rangle$
   $= \langle u_2, (0.84, 0.09, 0.01)\rangle;$
2. $e_1 + e_2 =$
   $\langle u_1 + u_2, (0.6 + 0.4 - 0.6 \cdot 0.4,\ 0.3 \cdot 0.3,\ 0.1 \cdot 0.3)\rangle$
   $= \langle u_3, (0.76, 0.09, 0.03)\rangle;$
3. $e_1 \times e_2 =$
   $\langle u_1 \times u_2, (0.6 \cdot 0.4,\ 0.3 \cdot 0.3,\ 0.1 + 0.3 - 0.1 \cdot 0.3)\rangle$
   $= \langle u_2, (0.24, 0.09, 0.37)\rangle;$
4. $(e_1)^2 = \left\langle (u_1)^2, \left((0.6)^2,\ (0.3)^2,\ 1-(1-0.1)^2\right)\right\rangle$
   $= \langle u_{1,} (0.36, 0.09, 0.19)\rangle.$

3.4 Theorem

If $e_j = \langle u_{e_j}, (P_{e_j}, I_{e_j}, N_{e_j})\rangle$, $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k})\rangle$ and $e_l = \langle u_{e_l}, (P_{el}, I_{e_l}, N_{e_l})\rangle$ be any three PFLNs and $\tau \ge 0$. Then the following identities are satisfying obviously.

1. $e_j + e_k = e_k + e_j;$
2. $e_j \times e_k = e_k \times e_j;$
3. $(e_j + e_k) + e_l = e_j + (e_k + e_l);$
4. $(e_j \times e_k) \times e_l = e_j \times (e_k \times e_l);$
5. $\tau e_j + \tau e_k = \tau(e_j + e_k),\ \tau \ge 0;$
6. $\tau_j e_j + \tau_k e_j = (\tau_j + \tau_k) e_j,\ \tau_j \ge 0\ \&\ \tau_k \ge 0;$
7. $(e_j \times e_k)^\tau = e_j^\tau \times e_k^\tau,\ \tau \ge 0;$
8. $e_j^{\tau_j} \times e_j^{\tau_k} = e_j^{\tau_j + \tau_k},\ \tau_j \ge 0\ \&\ \tau_k \ge 0.$

4. Comparison Rules for PFLNs

In this section, some functions which play important role for the ranking of PFLNs are described.

4.1 Definition

Let $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k})\rangle$ be any PFLN. Then

1. $sc(e_k)=\frac{(P_{e_k}+1-I_{e_k}+1-N_{e_k})}{3}\times\frac{u_{e_k}}{2}=\frac{1}{6}(2+P_{e_k}-I_{e_k}-N_{e_k})\times u_{e_k}$

which denoted as a score linguistic function.

2. $ac(e_k)=(P_{e_k}-N_{e_k})\times\frac{u_{e_k}}{2}=\frac{1}{2}(P_{e_k}-N_{e_k})\times u_{e_k}$ which denoted as an accuracy linguistic function.
3. $cr(e_k)=\frac{1}{2}(P_{e_k}\times u_{e_k})$ which denoted as a certainty linguistic function.

Idea takes from Definition 4.1, is the technique which using for equating the PFLNs can be described as

4.2 Definition

Let $e_j=\langle u_{e_j},(P_{e_j},I_{e_j},N_{e_j})\rangle$ and $e_k=\langle u_{e_k},(P_{e_k},I_{e_k},N_{e_k})\rangle$ be any two PFLNs. Then by using the Definition 4.1, equating technique can be described as,

a. If $sc(e_j)>sc(e_k)$, then $e_j>e_k$.

b. If $sc(e_j)=sc(e_k)$, and $ac(e_j)>ac(e_k)$, then $e_j>e_k$.

c. If $sc(e_j)=sc(e_k)$, $ac(e_j)=ac(e_k)$ and $cr(e_j)>cr(e_k)$, then $e_j>e_k$.

d. If $sc(e_j)=sc(e_k)$, $ac(e_j)=ac(e_k)$ and $cr(e_j)=cr(e_k)$, then $e_j=e_k$.

5. Weighted Aggregated operators for PFLNs

A wide range of linguistic aggregation operators have been proposed to aggregate the linguistic information. The present study proposes the PFLNWAA operator and PFLNWGA operator. These operators are utilized to aggregate the picture linguistic fuzzy information. These operators can be defined as follows.

5.1 Definition

Let $e_k=\langle u_{e_k},(P_{e_k},I_{e_k},N_{e_k})\rangle,\ (k\in N)$ be any collection of PFLNs and $PFLNWAA: PFLN^n\to PFLN$, then PFLNWAA describe as,

$$PFLNWAA(e_1,e_2,\ldots,e_n)=\sum_{k=1}^{n}\tau_k e_k,$$

In which $\tau=\{\tau_1,\tau_2,\ldots,\tau_n\}$ be the weight vector $e_k=\langle u_{e_k},(P_{e_k},I_{e_k},N_{e_k})\rangle$, $k\in N$, with $\tau_k\geq 0$ and $\Sigma_{k=1}^{n}\tau_k=1$.

5.2 Theorem

Let $e_k=\langle u_{e_k},(P_{e_k},I_{e_k},N_{e_k})\rangle,\ (k\in N)$ be any collection of PFLNs. Then by utilizing the Definition 3.2 and operational properties of PFLNs, we have obtained the following outcome.

$$PFLNWAA(e_1,e_2,\ldots,e_n)=$$
$$\left\langle u_{\sum_{k=1}^{n}\tau_k e_k},(1-\Pi_{k=1}^{n}(1-P_{e_k})^{\tau_k},\Pi_{k=1}^{n}I_{e_k}^{\tau_k},\Pi_{k=1}^{n}N_{e_k}^{\tau_k})\right\rangle,$$

Where $\tau=\{\tau_1,\tau_2,\ldots,\tau_n\}$ be the weight vector $e_k=\langle u_{e_k},(P_{e_k},I_{e_k},N_{e_k})\rangle$, $k\in N$, with $\tau_k\geq 0$ and $\Sigma_{k=1}^{n}\tau_k=1$.

Proof

We shall prove the result by using the principle of mathematical induction on k. Since $e_k=\langle u_{e_k},(P_{e_k},I_{e_k},N_{e_k})\rangle$, $k\in N$ be the collection of PFSs. Then, the following steps of the mathematical induction have been followed

(a) For $n=2$, since

$$\tau_1 e_1=\left\langle u_{\tau_1 e_1},(1-(1-P_{e_1})^{\tau_1},I_{e_1}^{\tau_1},N_{e_1}^{\tau_1})\right\rangle$$

and

$$\tau_2 e_2=\left\langle u_{\tau_2 e_2},(1-(1-P_{e_2})^{\tau_2},I_{e_2}^{\tau_2},N_{e_2}^{\tau_2})\right\rangle$$

Then

$$PFLNWAA(e_1,e_2)=\tau_1 e_1+\tau_2 e_2$$
$$=\left\langle u_{\tau_1 e_1},(1-(1-P_{e_1})^{\tau_1},I_{e_1}^{\tau_1},N_{e_1}^{\tau_1})\right\rangle+\left\langle u_{\tau_2 e_2},(1-(1-P_{e_2})^{\tau_2},I_{e_2}^{\tau_2},N_{e_2}^{\tau_2})\right\rangle$$

$$=\left\langle u_{\tau_1 e_1}+u_{\tau_2 e_2},\begin{pmatrix}((1-(1-P_{e_1})^{\tau_1})+(1-(1-P_{e_2})^{\tau_2})\\ -(1-(1-P_{e_1})^{\tau_1})\cdot(1-(1-P_{e_2})^{\tau_2})),\\ (I_{e_1}^{\tau_1}\cdot I_{e_2}^{\tau_2}),(N_{e_1}^{\tau_1}\cdot N_{e_2}^{\tau_2})\end{pmatrix}\right\rangle$$

$$=\left\langle u_{\tau_1 e_1+\tau_2 e_2},\begin{pmatrix}(2-(1-P_{e_1})^{\tau_1}-(1-P_{e_2})^{\tau_2}-1+\\ (1-P_{e_1})^{\tau_1}+(1-P_{e_2})^{\tau_2}\\ -(1-P_{e_1})^{\tau_1}\cdot(1-P_{e_2})^{\tau_2}),\\ (I_{e_1}^{\tau_1}\cdot I_{e_2}^{\tau_2}),\ (N_{e_1}^{\tau_1}\cdot N_{e_2}^{\tau_2})\end{pmatrix}\right\rangle$$

$$=\left\langle u_{\tau_1 e_1+\tau_2 e_2},\begin{pmatrix}1-(1-P_{e_1})^{\tau_1}\cdot(1-P_{e_2})^{\tau_2},\\ I_{e_1}^{\tau_1}\cdot I_{e_2}^{\tau_2},\ N_{e_1}^{\tau_1}\cdot N_{e_2}^{\tau_2}\end{pmatrix}\right\rangle$$

$$=\left\langle u_{\sum_{k=1}^{2}\tau_k e_k},(1-\Pi_{k=1}^{2}(1-P_{e_k})^{\tau_k},\ \Pi_{k=1}^{2}I_{e_k}^{\tau_k},\ \Pi_{k=1}^{2}N_{e_k}^{\tau_k})\right\rangle$$

(b) Suppose that outcome is true for $n = z$ that is,

$$PFLNWAA(e_1, e_2, \ldots, e_z) = \left\langle \begin{array}{l} u_{\sum_{k=1}^{z}\tau_k e_k}, (1-\Pi_{k=1}^{z}(1-P_{e_k})^{\tau_k}, \\ \Pi_{k=1}^{z} I_{e_k}^{\tau_k}, \Pi_{k=1}^{z} N_{e_k}^{\tau_k}) \end{array} \right\rangle,$$

(c) Now we should prove that outcome is true for $n = z+1$, by utilizing the (a) and (b) we have:

$$PFLNWAA(e_1, e_2, \ldots, e_z, e_{z+1}) = \sum_{k=1}^{z} \tau_k e_k + \tau_{z+1} e_{z+1}$$

$$= \left\langle u_{\sum_{k=1}^{z}\tau_k e_k}, (1-\Pi_{k=1}^{z}(1-P_{e_k})^{\tau_k}, \Pi_{k=1}^{z} I_{e_k}^{\tau_k}, \Pi_{k=1}^{z} N_{e_k}^{\tau_k}) \right\rangle + \left\langle u_{\tau_{z+1} e_{z+1}}, (1-(1-P_{e_{z+1}})^{\tau_{z+1}}, I_{e_{z+1}}^{\tau_{z+1}}, N_{e_{z+1}}^{\tau_{z+1}}) \right\rangle$$

$$= \left\langle u_{\sum_{k=1}^{z+1}\tau_k e_k}, (1-\Pi_{k=1}^{z+1}(1-P_{e_k})^{\tau_k}, \Pi_{k=1}^{z+1} I_{e_k}^{\tau_k}, \Pi_{k=1}^{z+1} N_{e_k}^{\tau_k}) \right\rangle,$$

i.e., outcome is satisfying for $n = z+1$. Thus, outcome is satisfied for whole n. Therefore,

$$PFLNWAA(e_1, e_2, \ldots, e_n) = \left\langle u_{\sum_{k=1}^{n}\tau_k e_k}, (1-\Pi_{k=1}^{n}(1-P_{e_k})^{\tau_k}, \Pi_{k=1}^{n} I_{e_k}^{\tau_k}, \Pi_{k=1}^{n} N_{e_k}^{\tau_k}) \right\rangle$$

We prove this.

5.3 Properties

There are some properties which are fulfilled by the PFLNWAA operator obviously.

(a) Idempotency:

Let $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $(k \in N)$ be any collection of PFLNs. If all of $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $(k \in N)$ are identical. Then

$$PFLNWAA(e_1, e_2, \ldots, e_n) = e.$$

(b) Boundedness:

Let $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $(k \in N)$ be any collection of PFLNs. Assume that $e_k^- = \langle \min_k u_{e_k}, (\min_k P_{e_k}, \min_k I_{e_k}, \max_k N_{e_k}) \rangle$ and $e_k^+ = \langle \max_k u_{e_k}, (\max_k P_{e_k}, \min_k I_{e_k}, \min_k N_{e_k}) \rangle$ for all $k \in N$, therefore

$$e_k^- \subseteq PFLNWAA(e_1, e_2, \ldots, e_n) = e_k^+.$$

(c) Monotonically:

Let $ȩ_k = \langle u_{ȩ_k}, (P_{ȩ_k}, I_{ȩ_k}, N_{ȩ_k}) \rangle$, $(k \in N)$ be any collection of PFLNs. If it satisfies that $e_k \subseteq ȩ_k$ for all, $k \in N$, then

$$PFLNWAA(e_1, e_2, \ldots, e_n) = PFLNWAA(ȩ_1, ȩ_2, \ldots, ȩ_n).$$

5.4 Definition

Let $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $(k \in N)$ be any collection of PFLNs and $PFLNWGA : PFLN^n \to PFLN$, then PFLNWGA operator is described as,

$$PFLNWGA(e_1, e_2, \ldots, e_n) = \prod_{k=1}^{n} e_k^{\tau_k},$$

In which $\tau = \{\tau_1, \tau_2, \ldots, \tau_n\}$ be the weight vector of $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $k \in N$, with $\tau_k \geq 0$ and $\sum_{k=1}^{n} \tau_k = 1$.

5.5 Theorem

Let $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $(k \in N)$ be any collection of PFLNs. Then by utilizing the Definition 5.1 and operational properties of PFLNs, we have obtained the following outcome.

$$PFLNWGA(e_1, e_2, \ldots, e_n) = \left\langle u_{\sum_{k=1}^{n} e_k^{\tau_k}}, \Pi_{k=1}^{n} P_{e_k}^{\tau_k}, \Pi_{k=1}^{n} I_{e_k}^{\tau_k}, (1-\Pi_{k=1}^{n}(1-N_{e_k})^{\tau_k} \right\rangle,$$

Where $\tau = \{\tau_1, \tau_2, \ldots, \tau_n\}$ be the weight vector $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $k \in N$, with $\tau_k \geq 0$ and $\sum_{k=1}^{n} \tau_k = 1$.

Proof:

Similar as Theorem 5.2, so procedure is eliminating here.

5.6 Properties

There are some properties which are fulfilled by the PFLNWGA operator obviously.

(a) Idempotency:

Let $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $(k \in N)$ be any collection of PFLNs. If all $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $(k \in N)$ are identical. Then

$$PFLNWGA(e_1, e_2, \ldots, e_n) = e.$$

(b) Boundedness:

Let $e_k = \langle u_{e_k}, (P_{e_k}, I_{e_k}, N_{e_k}) \rangle$, $(k \in N)$ be any collection of PFLNs. Assume that $e_k^- = \langle \min_k u_{e_k}, (\min_k P_{e_k}, \min_k I_{e_k}, \max_k N_{e_k}) \rangle$ and $e_k^+ = \langle \max_k u_{e_k}, (\max_k P_{e_k}, \min_k I_{e_k}, \min_k N_{e_k}) \rangle$ for all $k \in N$, then

$$e_k^- \subseteq PFLNWGA(e_1, e_2, \ldots, e_n) = e_k^+.$$

(c) Monotonically:

Let $ȩ_k = \langle u_{ȩ_k}, (P_{ȩ_k}, I_{ȩ_k}, N_{ȩ_k}) \rangle$, $(k \in N)$ be any collection of PFLNs. If it satisfies that $e_k \subseteq ȩ_k$ for all, $k \in N$, then

$$PFLNWGA(e_1, e_2, \ldots, e_n) = PFLNWGA(ȩ_1, ȩ_2, \ldots, ȩ_n).$$

6. MAGDM Method Utilizing the PFLNWAA and PFLNWGA Operators

This section proposes the technique to solve the MAGDM problems by utilizing the PFLNWAA and PFLNWGA operators. For a MAGDM problem, assuming that $C = \{c_1, c_2, \ldots, c_m\}$ be any finite collection of m alternatives, $G = \{g_1, g_2, \ldots, g_n\}$ be any finite collection of n attributes and $T = \{t_1, t_2, \ldots, t_p\}$ be any collection of p DMs. If the zth $(z = 1, 2, \ldots, p)$ DM deliver the assessment of the alternative $c_i (i = 1, 2, \ldots, m)$ on the attribute $g_i (i = 1, 2, \ldots, n)$ under any linguistic discrete term set. Let

$$B = [b_{jk}] = \left[\langle u_{e_{jk}}, (P_{e_{jk}}, I_{e_{jk}}, N_{e_{jk}}) \rangle\right]_{m \times n}$$

$$= \begin{bmatrix} \langle u_{e_{11}}, (P_{e_{11}}, I_{e_{11}}, N_{e_{11}}) \rangle & \langle u_{e_{12}}, (P_{e_{12}}, I_{e_{12}}, N_{e_{12}}) \rangle & \cdots & \langle u_{e_{1n}}, (P_{e_{1n}}, I_{e_{1n}}, N_{e_{1n}}) \rangle \\ \langle u_{e_{21}}, (P_{e_{21}}, I_{e_{21}}, N_{e_{21}}) \rangle & \langle u_{e_{22}}, (P_{e_{22}}, I_{e_{22}}, N_{e_{22}}) \rangle & \cdots & \langle u_{e_{2n}}, (P_{e_{2n}}, I_{e_{2n}}, N_{e_{2n}}) \rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle u_{e_{m1}}, (P_{e_{m1}}, I_{e_{m1}}, N_{e_{m1}}) \rangle & \langle u_{e_{m2}}, (P_{e_{m2}}, I_{e_{m2}}, N_{e_{m2}}) \rangle & \cdots & \langle u_{e_{mn}}, (P_{e_{mn}}, I_{e_{mn}}, N_{e_{mn}}) \rangle \end{bmatrix}$$

be the DM, where $\langle u_{e_{jk}}, (P_{e_{jk}}, I_{e_{jk}}, N_{e_{jk}}) \rangle$ are the collection of PFLNs and represents the evaluation information of every alternative $c_i (i = 1, 2, \ldots, m)$ on attribute $g_i (i = 1, 2, \ldots, n)$ with respect to the linguistic term $u_{e_{jk}}$. If $\tau = \{\tau_1, \tau_2, \ldots, \tau_n\}$ be the weight vector of attribute, with $\tau_k \geq 0$, $\sum_{k=1}^{n} \tau_k = 1$, and the weight vector of DMs is $Q = (q_1, q_2, \ldots, q_p)$, with $q_k \geq 0$ and $\Sigma_{k=1}^{p} q_k = 1$. Then, listed below the main technique of handling the MAGDM problems:

STEP-1. Normalized the given Decision Matrix.

In extensively, we have two kinds of criterion one is said to be positive criteria and other one said to be negative criteria. For uniform criterion, we need to modify the negative criteria into positive criteria. If criterion is uniform, then there is no need to be normalized.

STEP-2. Find out the comprehensive evaluation values for every alternative.

Utilizing the Theorem 5.2 and 5.4 to find out every value of the alternative c_i.

STEP-3. Find out the sc, ac and cr linguistic function values.

Calculating the score, accuracy and certainty linguistic function values respectively by utilizing the Definition 4.1.

STEP-4. Rank all the alternative.

By viewing the step-3 rank the all the alternative by utilizing the comparison technique in Definition 4.2. Then choose the best one(s).

6.1 A Descriptive Example

A brief illustrative example of the new approach in a linguistic decision-making problem is provided in this section.

Now we quote the example [18, 20, 22] whose describe the evaluation investment company to invest money in best choose. There are four manageable alternatives, (a) c_1 is car company; (b) c_2 is food company; (c) c_3 is a computer company; (d) c_4 is an arms company. According to the attributes company takes the decision, (a) g_1 is the risk; (b) g_2 is the growth; (c) g_3 is the environmental impact. The weight vector of the attributes is $\tau = (0.35, 0.25, 0.4)$. Where the evaluation information is denotes by the form of PFLNs under the linguistic term set S= { $u_0 =$ extremely poor, $u_1 =$ very poor, $u_2 =$ poor, $u_3 =$ medium, $u_4 =$ rich, $u_5 =$ very rich, $u_6 =$ extremely rich}. Now we can calculate the following picture fuzzy linguistic number decision matrix as

$$[b_{jk}] = \begin{bmatrix} \langle u_1, (0.6, 0.2, 0.2) \rangle & \langle u_2, (0.8, 0.1, 0.1) \rangle & \langle u_1, (0.6, 0.1, 0.3) \rangle \\ \langle u_2, (0.5, 0.3, 0.2) \rangle & \langle u_5, (0.5, 0.2, 0.3) \rangle & \langle u_3, (0.8, 0.1, 0.1) \rangle \\ \langle u_1, (0.4, 0.2, 0.4) \rangle & \langle u_3, (0.6, 0.3, 0.1) \rangle & \langle u_2, (0.4, 0.2, 0.4) \rangle \\ \langle u_1, (0.3, 0.1, 0.6) \rangle & \langle u_1, (0.7, 0.1, 0.2) \rangle & \langle u_4, (0.7, 0.1, 0.2) \rangle \end{bmatrix}$$

STEP-1

Since the attributes are uniform so there is no need to normalize.

STEP-2

Utilizing the Theorem 5.2 to find out every value of the alternative c_i as

$$\left[b_{jk}\right]_{AA}=\begin{bmatrix} e_1=\langle u_{1.25},(0.664,0.198,0.127)\rangle \\ e_2=\langle u_{3.15},(0.653,0.175,0.168)\rangle \\ e_3=\langle u_{1.9},(0.458,0.221,0.283)\rangle \\ e_4=\langle u_{1.8},(0.596,0.100,0.294)\rangle \end{bmatrix}$$

Utilizing the Theorem 5.4 to find out assessment values of the alternative c_i as,

$$\left[b_{jk}\right]_{GA}=\begin{bmatrix} e_1=\langle u_{1.25},(0.645,0.198,0.136)\rangle \\ e_2=\langle u_{3.15},(0.603,0.175,0.189)\rangle \\ e_3=\langle u_{1.9},(0.443,0.221,0.336)\rangle \\ e_4=\langle u_{1.8},(0.520,0.100,0.372)\rangle \end{bmatrix}$$

STEP-3

Now we find out the score, accuracy and certainty linguistic function values respectively by utilizing the Definition 4.1 as,

$$\underbrace{\begin{pmatrix} sc(e_1)=0.487 & ac(e_1)=0.336 & cr(e_1)=0.415 \\ sc(e_2)=1.212 & ac(e_2)=0.764 & cr(e_2)=1.028 \\ sc(e_3)=0.619 & ac(e_3)=0.166 & cr(e_3)=0.435 \\ sc(e_4)=0.661 & ac(e_4)=0.272 & cr(e_4)=0.536 \end{pmatrix}}_{\text{By using the PFLNWAA}}$$

and

$$\underbrace{\begin{pmatrix} sc(e_1)=0.482 & ac(e_1)=0.318 & cr(e_1)=0.403 \\ sc(e_2)=1.175 & ac(e_2)=0.652 & cr(e_2)=0.950 \\ sc(e_3)=0.597 & ac(e_3)=0.102 & cr(e_3)=0.421 \\ sc(e_4)=0.614 & ac(e_4)=0.133 & cr(e_4)=0.468 \end{pmatrix}}_{\text{By using the PFLNWGA}}$$

STEP-4.

Now rank the all the alternative by utilizing the comparison technique in Definition 4.2 for PFLNWAA operator as,

$$\begin{aligned} &sc(e_2)=1.212>sc(e_4)=0.661>sc(e_3) \\ &=0.619>sc(e_1)=0.487 \end{aligned}$$

so, by utilizing the Definition 4.2, we obtained the result which is

$$e_2>e_4>e_3>e_1$$

and by using the PFLNWAA, e_2 is the best choose.

Now rank the all the alternative by utilizing the comparison technique in Definition 4.2 for PFLNWGA operator as,

$$\begin{aligned} &sc(e_2)=1.175>sc(e_4)=0.614>sc(e_3) \\ &=0.597>sc(e_1)=0.482 \end{aligned}$$

So, by utilizing the Definition 4.2, we obtained the result which is

$$e_2>e_4>e_3>e_1$$

and by using the PFLNWGA, e_2 is the best choose.

7. Conclusion

Information aggregation process plays a vital role during the decision-making process and hence in this direction, the almost all the researchers have worked on the picture fuzzy set by considering the degrees of the positive membership, neutral membership and negative membership only. But, it has been observed that in some situations, like in case of voting, human opinions involving more answers of the types: yes, abstain, no, refusal, which cannot be accurately represented in numerically. For this, picture linguistic fuzzy set, which is an extension of the intuitionistic linguistic fuzzy set, has been used in the manuscript, and correspondingly aggregation operators have been defined. Various desirable properties of these operators have also been investigated in detail. Finally, based on these operators, a decision-making method has been proposed for ranking the different alternatives by using picture linguistic fuzzy information. The approach has been illustrated with a numerical example for showing their effectiveness as well as stability.

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